

① Distinguish between scalar and vector.

A scalar is a quantity that has only magnitude. Quantity such as time, mass, distance, temperature, electric potential, and population are scalars.

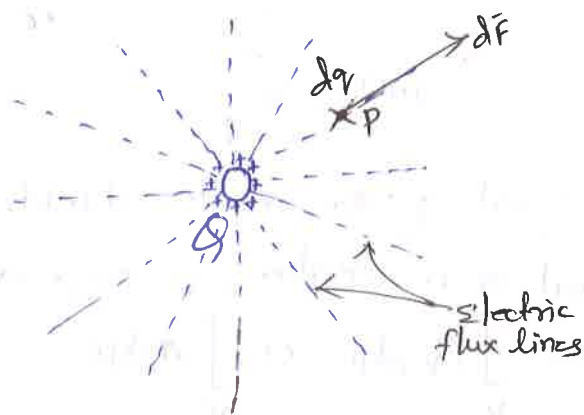
A vector is a quantity that has both magnitude and direction. vector quantities include velocity, force, displacement, and electric field intensity.

② Define electromagnetic field.

This is a region of space in which electric and magnetic fields exist simultaneously. such a field is always a varying field which varies with respect to time, as in the case of Radio wave, T.V wave, mobile phone system etc.

③ Define electric field.

- * Electric field is produced by charged body.
- * This is a region of space in which electric charge experience force.
- * Electric field can exist in free space, they can also exist in material media like glass, mica, paper, wood, glass, liquid gases etc.



$Q \rightarrow$ is charged body. The space surrounding this body is electric field.

$dq \rightarrow$ is a positive test charge having negligible charge in coulombs.

$dF \rightarrow$ is repulsive force experienced by test charge in newtons.

4 state the properties of electric flux lines.

- (i) These are continuous lines.
- (ii) These lines behave like stretched rubber bands.
- (iii) Flux lines do not cross each other.
- (iv) Tangent at any point on the flux line gives the direction of field at that point.
- (v) Electric flux line starts at +ve charge and ends on -ve charge.
- (vi) Flux lines contain energy.

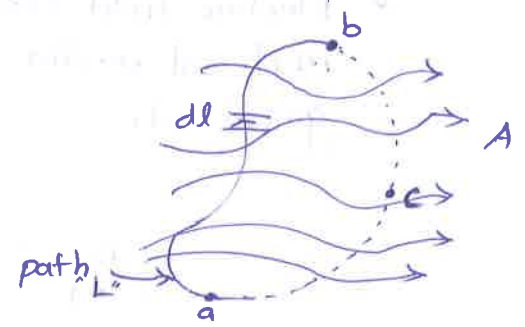
5 Mention the source of electromagnetic fields.

- (i) Transformer
- (ii) Electric Relay
- (iii) Radio/T.V
- (iv) Telephone
- (v) Electric motor
- (vi) Transmission lines
- (vii) wave guides
- (viii) communication satellites
- (ix) Radar
- (x) Laser.

6 Define the line integral of a vector field.

The line integral of vector 'A' along the path 'L' is given by $\int A \cdot dl$. If the path is closed, the line integral becomes the circulation of 'A' around L, that is

$$\oint_L A \cdot dl$$



7 Define the volume integral of the vector field.

The volume integral of a scalar P_v over a volume V is defined as $\int_V P_v dv$ or $\int_V A dv$

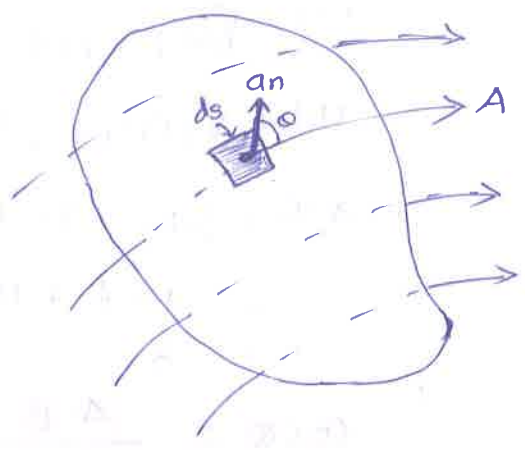
8 Define the surface integral of a vector field.

The surface integral of a vector 'A' across a surface 'S' is defined as $\int_S A \cdot ds$.

When the surface 'S' is closed, the surface integral becomes the net outward flux of 'A' across S, that is

$$\psi = \oint A \cdot ds$$

Where, at any point on 'S', an is the unit normal to 'S'.



9 Given $\bar{A} = 3\bar{u}_x + 4\bar{u}_y$ and $\bar{B} = 12\bar{u}_y - 5\bar{u}_z$, find the angle between them by using (a) The dot product; (b) The cross product.

(a) Dot product

$$A \cdot B = |A| \cdot |B| \cos \theta$$

$$\cos \theta = \frac{A \cdot B}{|A| \cdot |B|}$$

$$|A| = \sqrt{(3)^2 + (4)^2} = 5$$

$$|B| = \sqrt{(12)^2 + (-5)^2} = 13$$

$$\begin{aligned} A \cdot B &= (3\bar{u}_x + 4\bar{u}_y) \cdot (12\bar{u}_y - 5\bar{u}_z) \\ &= 3(0) + (4)(12) - 5(0) \\ &= 48 \end{aligned}$$

$$\cos \theta = \frac{A \cdot B}{|A| \cdot |B|} = \frac{48}{5 \times 13} = 0.7385$$

$$\theta = \cos^{-1}(0.7385)$$

$$\theta = 42.4^\circ$$

(b) Cross product

$$\bar{A} \times \bar{B} = |\bar{A}| \cdot |\bar{B}| \sin \theta$$

$$A \times B = \begin{vmatrix} \bar{u}_x & \bar{u}_y & \bar{u}_z \\ 3 & 4 & 0 \\ 0 & 12 & -5 \end{vmatrix}$$

$$\begin{aligned} &= \bar{u}_x(-20-0) - \bar{u}_y(-15-0) + \bar{u}_z(36-0) \\ &= -20\bar{u}_x + 15\bar{u}_y + 36\bar{u}_z \end{aligned}$$

$$|A \times B| = \sqrt{(-20)^2 + (15)^2 + (36)^2}$$

$$|A \times B| = \sqrt{1921} = 43.829$$

$$|A| = \sqrt{(3)^2 + (4)^2} = 5$$

$$|B| = \sqrt{(12)^2 + (-5)^2} = 13$$

$$\sin \theta = \frac{|A \times B|}{|A| \cdot |B|} = \frac{43.829}{65} = 0.6743$$

$$\theta = \sin^{-1}(0.6743)$$

$$\theta = 42.4^\circ$$

(note! same as above)

(10) Two vectorial quantities $\vec{A} = 4\vec{i} + 3\vec{j} + 5\vec{k}$ and $\vec{B} = \vec{i} - 2\vec{j} + 2\vec{k}$ are known to be oriented in two unique directions. Determine angular separation between them.

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$$

$$|\vec{A}| = \sqrt{(4)^2 + (3)^2 + (5)^2} = 7.07$$

$$|\vec{B}| = \sqrt{(1)^2 + (-2)^2 + (2)^2} = 3$$

$$\vec{A} \cdot \vec{B} = (4\vec{i} + 3\vec{j} + 5\vec{k}) \cdot (\vec{i} - 2\vec{j} + 2\vec{k})$$

$$= 4 - 6 + 10$$

$$= 8$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|} = \frac{8}{7.07 \times 3} = 0.377$$

$$\theta = \cos^{-1}(0.377)$$

$$\theta = 67.84^\circ$$

(11) state the condition for a vector \vec{A} to be (a) solenoidal (b) irrotational.

(a) For solenoidal.

$$\nabla \cdot \vec{A} = 0 \text{ (dot product)}$$

(b) For irrotational

$$\nabla \times \vec{A} = 0 \text{ (cross product)}$$

(12) what are the conditions for two vector \vec{A} and \vec{B} to be (i) parallel (ii) perpendicular.

(i) For parallel condition

$$\vec{A} \times \vec{B} = 0 \text{ (cross product)}$$

(ii) perpendicular condition

$$\vec{A} \cdot \vec{B} = 0 \text{ (dot product)}$$

- 13) show that the two vectors $\vec{A} = 6\vec{i} + \vec{j} - 5\vec{k}$ and $\vec{B} = 3(\vec{i} - \vec{j} + \vec{k})$ are perpendicular to each other. (5)

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (6\vec{i} + \vec{j} - 5\vec{k}) \cdot (3\vec{i} - 3\vec{j} + 3\vec{k}) \\ &= 6(3) + 1(-3) + (-5)(3) \\ &= 18 - 3 - 15 \\ &= 0\end{aligned}$$

∴ The two vectors are perpendicular.

- 14) show that vector field $\vec{v} = (x+3y)\vec{i} + (y-3z)\vec{j} + (x-2z)\vec{k}$ is solenoidal.

$$\nabla \cdot \vec{v} = \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \cdot (x+3y)\vec{i} + (y-3z)\vec{j} + (x-2z)\vec{k}$$

$$\nabla \cdot \vec{v} = \frac{\partial}{\partial x} (x+3y) + \frac{\partial}{\partial y} (y-3z) + \frac{\partial}{\partial z} (x-2z)$$

$$\nabla \cdot \vec{v} = 1 + 1 - 2$$

$$\nabla \cdot \vec{v} = 0$$

∴ $\nabla \cdot \vec{v}$ is solenoidal.

- 15) show that the vector $\vec{H} = 3y^4 z^2 \vec{a}_x + 4x^3 z^2 \vec{a}_y + 3x^2 y^2 \vec{a}_z$ is solenoidal.

$$\nabla \cdot \vec{H} = \left(\frac{\partial}{\partial x} \vec{a}_x + \frac{\partial}{\partial y} \vec{a}_y + \frac{\partial}{\partial z} \vec{a}_z \right) \cdot (3y^4 z^2 \vec{a}_x + 4x^3 z^2 \vec{a}_y + 3x^2 y^2 \vec{a}_z)$$

$$\nabla \cdot \vec{H} = \frac{\partial}{\partial x} (3y^4 z^2) + \frac{\partial}{\partial y} (4x^3 z^2) + \frac{\partial}{\partial z} (3x^2 y^2)$$

$$\nabla \cdot \vec{H} = 0 + 0 + 0$$

Hence \vec{H} is a solenoidal vector.

- 16) point 'P' and 'Q' are located at (0, 2, 4) and (-3, 1, 5). Calculate the distance between 'P' and 'Q'.

note: $\begin{matrix} x_1 & y_1 & z_1 \\ (0, & 2 & 4) \end{matrix} \begin{matrix} x_2 & y_2 & z_2 \\ (-3, & 1 & 5) \end{matrix}$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(0 - (-3))^2 + (2 - 1)^2 + (4 - 5)^2}$$

$$d = \sqrt{(-3)^2 + (1)^2 + (-1)^2}$$

$$d = \sqrt{9 + 1 + 1}$$

$$d = 3.3166 \text{ meter.}$$

(17) Given $A = 4a_x + 6a_y - 2a_z$ and $B = -2a_x + 4a_y + 8a_z$. show that the vectors are orthogonal.

Solution:

$$A \cdot B = |A| \cdot |B| \cos \theta$$

$$\cos \theta = \frac{A \cdot B}{|A| \cdot |B|}$$

$$|A| = \sqrt{(4)^2 + (6)^2 + (-2)^2} = 7.48$$

$$|B| = \sqrt{(-2)^2 + (4)^2 + (8)^2} = 9.1652$$

$$A \cdot B = (4a_x + 6a_y - 2a_z) \cdot (-2a_x + 4a_y + 8a_z)$$

$$A \cdot B = -8 + 24 - 16$$

$$A \cdot B = 0$$

$$\cos \theta = \frac{A \cdot B}{|A| \cdot |B|} = \frac{0}{(7.48) \cdot (9.1652)} = \frac{0}{68.556} = 0$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = 90^\circ$$

Hence a given vector is a orthogonal (90°)

(18) state the different types of co-ordinate systems.

(i) Rectangular (or) cartesian co-ordinates

(ii) cylindrical co-ordinates.

(iii) spherical co-ordinates.

(19) write down expression for different volume in
(i) cartesian (ii) spherical co-ordinate systems.

(i) In cartesian system $dV = dx dy dz$

(ii) In cylindrical co-ordinate system, $dV = r^2 \sin \theta dr d\theta d\phi$.

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write expression for differential length in (i) cartesian (ii) cylindrical and (iii) spherical co-ordinates.

(i) In cartesian system, $dl = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

(ii) In cylindrical system, $dl = \sqrt{(dp)^2 + (p d\phi)^2 + (dz)^2}$

(iii) In spherical co-ordinate system, $dl = \sqrt{(dr)^2 + (r d\theta)^2 + (r \sin\theta d\phi)^2}$

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How are the unit vector defined in cylindrical co-ordinate systems?

The unit vector in the cylindrical co-ordinate system are functions of position. It is convenient to express them in terms of the cylindrical coordinates and the unit vectors of the rectangular co-ordinate system which are not themselves functions of position.

$$\hat{\rho} = \frac{\vec{\rho}}{\rho} = \frac{x\hat{x} + y\hat{y}}{\rho} = \hat{x} \cos\phi + \hat{y} \sin\phi$$

$$\hat{\phi} = \hat{z} \times \hat{\rho} = -\hat{x} \sin\phi + \hat{y} \cos\phi$$

$$\hat{z} = \hat{z}$$

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Give the spherical co-ordinates of the point whose cartesian co-ordinates are $x=3; y=4; z=5$ units.

$$x=3; y=4; z=5$$

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 5^2} = 7.07 \text{ units}$$

$$\theta = \cos^{-1}(z/\rho) = \cos^{-1}(5/7.07) = 45^\circ$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(4/3) = 53.13^\circ$$

(23)

Express in matrix form the unit vector transformation from the rectangular to cylindrical co-ordinate system. ⑧

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

(24)

Transform a vector $\bar{A} = y\bar{i}_x + x\bar{i}_y + z\bar{i}_z$ in to cylindrical co-ordinates.

Solution:

$$\bar{A} = y\bar{i}_x + x\bar{i}_y + z\bar{i}_z$$

The rectangular co-ordinates are (y, x, z)

The cylindrical co-ordinates are (ρ, ϕ, z)

$$\begin{aligned} A_\rho &= A_x \cos\phi + A_y \sin\phi \\ &= y \cos\phi + (-x \sin\phi) \\ &= y \cos\phi - x \sin\phi \end{aligned}$$

where, $\phi = \tan^{-1}(y/x)$

But

$$x = \rho \cos\phi \quad \text{and} \quad y = \rho \sin\phi$$

$$A_\rho = \rho \sin\phi \cos\phi - \rho \cos\phi \sin\phi = 0$$

$$\begin{aligned} A_\phi &= -A_x \sin\phi + A_y \cos\phi \\ &= -y \sin\phi - x \cos\phi \end{aligned}$$

$$= -[\rho \sin\phi \cdot \sin\phi + \rho \cos\phi \cos\phi]$$

$$= -\rho [\sin^2\phi + \cos^2\phi]$$

$$= -\rho$$

$$A_z = z$$

\therefore Vector in cylindrical co-ordinates are

$$A = -\rho\bar{i}_\phi + z\bar{i}_z$$

(25) write poisson's equation in cylindrical co-ordinate system. (9)

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \left(\frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho}{\epsilon_0 \epsilon_1}$$

(26) write the poisson's equation in spherical co-ordinate system.

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = \frac{-\rho}{\epsilon_0 \epsilon_1}$$

(27) write the poisson's equation in cartesian co-ordinate system.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho}{\epsilon_0 \epsilon_1}$$

(28) Define Gradient.

Gradient means slope (or) rate of change.

The gradient of any scalar function is the maximum space rate of change of that function. If the scalar 'V' represents electric potential then ∇V represents potential gradient.

$$\nabla V = \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k}$$

(29) How can a vector field be expressed as gradient of scalar field?

We know the well known relation

$$\vec{E} = -\nabla \phi$$

where,

\vec{E} is vector field quantity and
 $\nabla \phi$ is gradient of electric potential ϕ
 ϕ is a scalar field quantity.

30 Explain the physical significance of ∇V .

∇V is gradient of 'V' and it means the rate of change of 'V' in x, y, z direction.

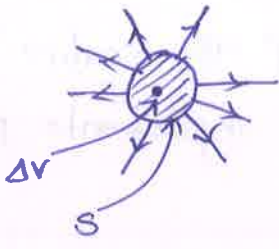
\therefore In general

$$\nabla V = \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k}$$

31 Define divergence and its physical meaning.

The divergence of vector 'F' is a scalar quantity a vector 'F' at a given point 'P' is the outwards flux per unit volume as the volume shrinks about 'P'.

$$\text{Div } \vec{F} = \nabla \cdot \vec{F} = \lim_{\Delta V \rightarrow 0} \frac{\oiint_S \vec{F} \cdot d\vec{s}}{\Delta V} = \text{Flux per unit volume.}$$



$S \rightarrow$ is a closed surface
 $\Delta V \rightarrow$ volume enclosed by S and also this is infinitesimally small volume.

32 Find the divergence of the vector function $\vec{D} = e^{-x} \sin y \hat{a}_x - e^{-x} \cos y \hat{a}_y + 2z \hat{a}_z$.

$$\begin{aligned} \nabla \cdot \vec{D} &= \text{div } \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \\ &= -e^{-x} \sin y + e^{-x} \sin y + 2 \\ &= 2 \end{aligned}$$

33) State Divergence theorem.

According to this theorem, the volumetric integral of divergence of a given vector function carried out over a specified volume is equal to the surface integral of the same vector function carried out over the closed surface which encloses the given volume.

This can be expressed as

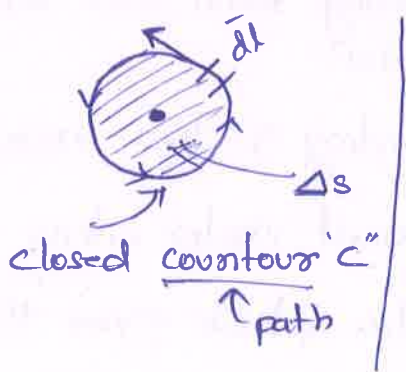
$$\iiint_V \text{Div } \vec{F} \, dV = \oiint_S \vec{F} \cdot d\vec{s}$$

- $V \rightarrow$ volume of any shape
- $S \rightarrow$ closed surface of the entire volume
- $d\vec{s} \rightarrow$ small surface area direction is outward normal
- $dV \rightarrow$ Differential volume (very very small volume)

34) Define CURL.

By definition curl of a vector field \vec{A} at any given point is

$$\text{Curl } \vec{A} = \lim_{\Delta s \rightarrow 0} \frac{\oint_C \vec{A} \cdot d\vec{l}}{\Delta s} \vec{a}_n$$



- where,
- $\Delta s \rightarrow$ is infinitesimally small area which enclosed the given point
 - $C \rightarrow$ is the boundary of this area, 'c' has got a direction
 - $d\vec{l} \rightarrow$ is displacement vector along the path 'c'.
 - $\vec{a}_n \rightarrow$ is unit vector which gives the direction of curl \vec{A} .

note: If $\text{Curl } \vec{A} = 0$ then the vector \vec{A} is said to be irrotational, (or) conservative.

35) prove that $\text{curl grad } \phi = 0$

$$\text{Grad } \phi = \nabla \phi = \left[\frac{\partial \phi}{\partial x} \bar{i} + \frac{\partial \phi}{\partial y} \bar{j} + \frac{\partial \phi}{\partial z} \bar{k} \right]$$

$$\text{curl Grad } \phi = \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{bmatrix}$$

$$\text{curl Grad } \phi = \bar{i} \left(\frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial z \partial y} \right) - \bar{j} \left(\frac{\partial^2 \phi}{\partial x \partial z} - \frac{\partial^2 \phi}{\partial z \partial x} \right) + \bar{k} \left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} \right)$$

$$\text{curl Grad } \phi = 0$$

36) State the physical significance of curl of a vector fields.

By definition curl of vector field \bar{A} at any given point is.

$$\nabla \times \bar{A} = \text{curl } \bar{A} = \lim_{\Delta s \rightarrow 0} \frac{\oint_C \bar{A} \cdot d\bar{l}}{\Delta s} \bar{a}_n$$

where,

$\Delta s \rightarrow$ is infinitesimally small area which enclosed the given point.

$C \rightarrow$ is the boundary of this area. (C has got direction)

$d\bar{l} \rightarrow$ is displacement vector along the path C .

$\bar{a}_n \rightarrow$ is unit vector which gives the direction of curl \bar{A} .

(37) State Stokes's theorem.

According to this theorem the surface integral of curl of any vector function over a given open surface is equal to the line integral of the given vector function along the boundary or (contour) of the given open surface.

$$\oint_C \vec{A} \cdot d\vec{l} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

where,

$S \rightarrow$ the given open surface and

$C \rightarrow$ the boundary or contour of the open surface.

(38) List out the vector field quantities in electromagnetics:

* There are four fundamental vector field quantities in electromagnetics:

Electric	(i) Electric field intensity	E	V/m (volts per meter)
	(ii) Electric flux density (or) (Electric displacement)	D	C/m ² (Coulomb per meter ²)
magnetic	(i) magnetic flux density	B	T (Tesla (or) volt-second per square meter)
	(ii) magnetic field intensity	H	A/m (Ampere per meter)

(39) State Helmholtz's theorem.

A vector field (vector point function) is determined to within an additive constant if both its divergence and its curl are specified everywhere.

(40) show that vector field $\vec{A} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational (conservative).

Solution:

We know that for a irrotational (conservative) field vector curl must be zero.

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$\nabla \times \vec{A} = \vec{i} \left(\frac{\partial}{\partial y} (xy) - \frac{\partial}{\partial z} (zx) \right) - \vec{j} \left(\frac{\partial}{\partial x} (xy) - \frac{\partial}{\partial z} (yz) \right) + \vec{k} \left(\frac{\partial}{\partial x} (zx) - \frac{\partial}{\partial y} (yz) \right)$$

$$\nabla \times \vec{A} = \vec{i} (x-x) - \vec{j} (y-y) + \vec{k} (z-z)$$

$$\nabla \times \vec{A} = \vec{i} (0) - \vec{j} (0) + \vec{k} (0)$$

$$\nabla \times \vec{A} = 0$$

Hence \vec{A} is irrotational.

(41) verify whether the vector field $\vec{F} = y^2z\vec{a}_x + z^2x\vec{a}_y + x^2y\vec{a}_z$ is irrotational.

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z & z^2x & x^2y \end{vmatrix}$$

$$\nabla \times \vec{F} = \vec{i} \left(\frac{\partial}{\partial y} (x^2y) - \frac{\partial}{\partial z} (z^2x) \right) - \vec{j} \left(\frac{\partial}{\partial x} (x^2y) - \frac{\partial}{\partial z} (y^2z) \right) + \vec{k} \left(\frac{\partial}{\partial x} (z^2x) - \frac{\partial}{\partial y} (y^2z) \right)$$

$$\nabla \times \vec{F} = \vec{i} (x^2 - 2zx) - \vec{j} (2xy - y^2) + \vec{k} (z^2 - 2yz)$$

It is clear that the RHS of the above equation is not zero, so this vector is not irrotational.

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① Define Electric field Intensity.

The electric field intensity (or electric field strength) E is the force per unit charge when placed in an electric field.

Thus,

$$\vec{E} = \frac{d\vec{F}}{dq} \quad (\text{or}) \quad \vec{E} = \lim_{q \rightarrow 0} \frac{F}{Q}$$

or simply

$$\vec{E} = \frac{F}{Q}$$

$\vec{E} \rightarrow$ is a electric field intensity, is a vector quantity, and shows the strength (intensity) as well as direction of electric field at point 'p'.

② state coulomb's Law.

Coulomb's law states that the force 'F' between two point charges Q_1 and Q_2 is:

1. Along the line joining them
2. Directly proportional to the product Q_1 and Q_2 the charges.
3. Inversely proportional to the square of the distance 'd' between them.

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 \epsilon_r d^2} \quad \text{newtons (N)}$$

where,

 Q_1 and Q_2 are in coulombs (C) $\epsilon_0 \rightarrow$ is constant is known as permittivity of free space (F/m) $\epsilon_r \rightarrow$ Relative permittivity of the given medium. $d \rightarrow$ is the distance in meters (m)

③ Define Gauss's Law. (April/may 2023)

According to Gauss's law total electric flux coming out of a closed surface normally (perpendicular to the surface) is equal to net charge enclosed within the surface.

$$\text{i.e. } \Psi = Q$$

④ Name few applications of Gauss law in electrostatic

- (i) Electric field can be determined for shell
- (ii) Two concentric shell or cylinders.
- (iii) Gauss law is also applied to evaluate the electric field intensity in the closed surface, which is generally called as Gaussian surface, such that the electric field has a normal component which is a single fixed value zero at every point on the surface.

⑤ Give the significance of Gauss's law.

It simplifies a complicated problem into a simple problem.

⑥ Define electric flux density, \vec{D}

Electric flux density \vec{D} at any point in an electric field is defined as the number of flux lines passing through unit area normally.

$$\vec{D} = \frac{d\Psi}{dA} \text{ coulombs/m}^2$$

⑦ State the vector form of electric flux density.

$$\vec{D} = \epsilon_0 \epsilon_r \times \vec{E}$$

where, $\vec{D} \rightarrow$ is vector form of electric flux density
 $\vec{E} \rightarrow$ is electric field intensity in vector form.

⑧ Define electric potential.

③

Electric potential 'V' is defined as work done moving a unit positive charge from infinite up to the given point.

$$V = \frac{dw}{dq} \quad \text{Joules/Coulomb's (or) volts.}$$

⑨ The electric potential near the origin of a system of co-ordinates is $V = ax^2 + by^2 + cz^2$. Find the electric field at (1, 2, 3).

We know the relation $\vec{E} = -\nabla V$

$$\vec{E} = - \left[\frac{\partial V}{\partial x} \vec{i} + \frac{\partial V}{\partial y} \vec{j} + \frac{\partial V}{\partial z} \vec{k} \right]$$

$$\frac{\partial V}{\partial x} = 2ax + 0 + 0$$

$$\frac{\partial V}{\partial y} = 0 + 2by + 0$$

$$\frac{\partial V}{\partial z} = 0 + 0 + 2cz$$

$$\therefore \vec{E} = - \left[2ax \vec{i} + 2by \vec{j} + 2cz \vec{k} \right]$$

at the point (1, 2, 3), we have

$$\vec{E} = - \left[2a(1) \vec{i} + 2b(2) \vec{j} + 2c(3) \vec{k} \right]$$

$$\therefore \vec{E} = (-2a \vec{i} - 4b \vec{j} - 6c \vec{k})$$

⑩ Define potential difference.

potential difference is defined as the work done in moving a unit positive charge from one point to another point in an electric field.

- ⑪ Find the electric potential at a point (4,3) m due to a charge of 10^{-9} C located at the origin in free space. ④

$$\text{Formula is } V = \frac{Q}{4\pi \epsilon_0 \epsilon_r d} = \frac{10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times 1 \times d}$$

where,

$$d = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = 5$$

$$\therefore V = \frac{10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times 1 \times 5} = 1.7975 \text{ volts}$$

- ⑫ Explain why $\nabla \times E = 0$

Because it is conservative property of Electrostatic field.

- ⑬ what is meant by 'conservative property of electric field?'

The line integral of E along a closed path must be zero.

$$\oint_L E \cdot dl = 0 \rightarrow \textcircled{1}$$

$$\oint_L E \cdot dl = \int_S (\nabla \times E) \cdot ds = 0$$

(or)

$$\boxed{\nabla \times E = 0} \rightarrow \textcircled{2}$$

Any vector field that satisfies eqn ① & ② is said to be conservative or irrotational.

- (14) Distinguish between absolute electric potential and Relative electric potential. (5)

Absolute electric potential:- Absolute potential at any point in electric field is the potential at that point with respect to infinity.

Relative electric potential:- Relative potential is the potential at one point with respect to another point within the given electric field.

- (15) State the boundary field conditions at the interface between two perfect dielectrics.

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

- (16) Calculate the capacitance of a parallel plate capacitor having an electrode area of 100 cm^2 . The distance between the electrodes is 3 mm and the dielectric used has a permittivity of 3.6 the applied potential is 80 V . Also compute the charge on the plates.

$$C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 3.6 \times 0.01}{3.6 \times 10^{-3}}$$

$$C = \frac{3.18744 \times 10^{-3}}{3.6 \times 10^{-3}} = 88.54 \times 10^{-12}$$

$$C = 88.54 \text{ pf.}$$

(17) Define electric scalar potential.

A scalar potential 'V' is defined as work done moving a unit positive charge from infinite up to the given point in an electric field.

$$V = \frac{dw}{dq} \quad \text{Joules/Coulombs (or) volts}$$

(18) Determine the potential difference between the points 'a' and 'b' which are at distance of 0.5m and 0.1m respectively from a negative charge of 2.0×10^{-10} coulomb.

Solution:

$$x_2 = a = 0.5 \text{ m}$$

$$x_1 = b = 0.1 \text{ m}$$

note: $x \rightarrow$ is distance in meters

$$Q = -2 \times 10^{-10} \text{ coulomb}$$

$$V_a = \frac{Q}{4\pi\epsilon_0 x_1} = \frac{-2 \times 10^{-10}}{4\pi \times 8.854 \times 10^{-12} \times 0.5} = -3.597 \text{ volts}$$

$$V_b = \frac{Q}{4\pi\epsilon_0 x_2} = \frac{-2 \times 10^{-10}}{4\pi \times 8.854 \times 10^{-12} \times 0.1} = -17.985 \text{ volts.}$$

$$V = V_a - V_b$$

$$V = -3.597 - (-17.985)$$

$$V = 14.4 \text{ volts.}$$

(19) Find the electric field intensity at a distance of 20cm a charge of $2\mu\text{C}$ in vacuum.

Solution:

$$r = 20 \text{ cm} = 0.2 \text{ m}$$

$$Q = 2 \times 10^{-6} \text{ coulomb}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{2 \times 10^{-6}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times 0.2^2} = 449387.1219 \text{ Volts/m}$$

$$E = \frac{449387.1219}{1000} = 449.38 \text{ KV/m}$$

- (20) A uniform spherical volume charge distribution contains a total charge $9 \times 10^8 \text{ C}$ if the radius of this spherical volume is $2 \times 10^{-2} \text{ m}$. Find the volume charge density ρ .

Solution:

$$Q = 9 \times 10^8 \text{ C}$$

$$[\text{note: } V = \frac{4}{3} \pi r^3 \text{ in meters}]$$

$$r = 2 \times 10^{-2} \text{ m}$$

$$\rho = \frac{Q}{V} = \frac{9 \times 10^8}{\frac{4}{3} \pi r^3} \text{ C/m}^3 = \frac{9 \times 10^8}{\frac{4}{3} \times 3.14 \times (2 \times 10^{-2})^3}$$

$$\rho = 0.2985 \times 10^{-3} \text{ C/m}^3$$

- (21) What is the absolute potential at a point 'P' which is 2m from a point charge $Q = 5 \mu\text{C}$ and also find the work required to move a 8 nC charge from infinite to P.

Solution:

$$r = 2 \text{ m}, \quad Q = 5 \mu\text{C}$$

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{5 \times 10^{-6}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times 2}$$

$$V = 22480.753 \text{ volts}$$

(or)

$$V = 22.481 \times 10^3 \text{ volts}$$

$$V = \frac{\text{work done}}{\text{charge}}$$

$$\text{work done} = V \times \text{charge}$$

$$= 22.481 \times 10^3 \times 8 \times 10^{-9}$$

$$= 180 \mu\text{J}$$

- (22) A uniform line charge, infinite in extent, with $\rho_l = 20 \text{ nC/m}$ lies along the z-axis. Find E at $(6, 8, 3)$

Solution:

$$r = |r| = \sqrt{x^2 + y^2 + z^2} = \sqrt{6^2 + 8^2 + 3^2} = 10.441$$

$$E = \frac{\rho_l}{2\pi\epsilon_0 r} = \frac{20 \times 10^{-9}}{2 \times 3.14 \times 8.854 \times 10^{-12} \times 10.441} = 34.48 \text{ V/m}$$

23 what is the capacitance in a coaxial cable?

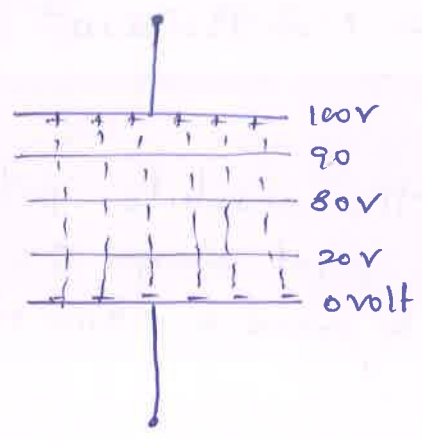
capacitance of a coaxial cable is given by

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln\left(\frac{R}{r}\right)} \text{ farads/meter}$$

24 Draw the equipotential lines and electric field lines for a parallel plate capacitor.

⇒ The vertical dotted lines are electric flux lines.

⇒ The horizontal lines are equipotential lines.

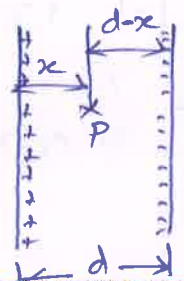


25 what is the practical significance of Dielectric strength?

The dielectric strength of insulation in electrical equipment is the deciding factor for determining the operating voltage of the equipment. If the dielectric strength is more we can operate the equipment at higher voltage.

26 what is the potential at a point 'p' due to two parallel line charges which are equal and opposite?

potential at point 'p' may be
 Expressed as $V = \frac{q}{2\pi \epsilon_0 \epsilon_r} [\ln x - \ln x] + \frac{-q}{2\pi \epsilon_0 \epsilon_r} [\ln x - \ln (d-x)]$ volts



(27)

Express Laplace and Poisson's equations.

Laplace equation is $\nabla^2 V = 0$ Poisson's equation is $\nabla^2 V = -\frac{\rho}{\epsilon_0 \epsilon_r}$

(28)

Calculate the capacitance of a parallel plate capacitor having an electrode area of 100 cm^2 . The distance between the electrode is 4 mm and the dielectric used has a permittivity of 3.5 . The applied potential is 100 volts .

Solution:

$$A = 100 \text{ cm}^2 = \frac{100}{100 \times 100} = 0.01 \text{ m}^2$$

$$\epsilon_r = 3.5$$

$$t = 4 \text{ mm} = \frac{4}{1000} = 0.004 \text{ m}$$

$$C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{\epsilon_0 \epsilon_r A}{t} = \frac{8.854 \times 10^{-12} \times 3.5 \times 0.01^2}{0.004}$$

$$C = \frac{2.83328 \times 10^{-13}}{0.004} = 7.0832 \times 10^{-11} = 70.832 \times 10^{-12}$$

$$C = 70.832 \text{ pf}$$

(29) Determine the capacitance of a parallel plate capacitor composed of tin foil sheets, 25 cm square plate separated through a glass which is having a relative permittivity of 6 . The distance between the plates is 0.5 cm .

Solution:

$$A = 25 \text{ cm}^2 = \frac{25}{100 \times 100} = 0.0025 \text{ m}^2$$

$$\epsilon_r = 6, \quad t = 0.5 \text{ cm} = \frac{0.5}{100} = 5 \times 10^{-3} \text{ m}$$

$$C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 6 \times 0.0025^2}{5 \times 10^{-3}}$$

$$C = \frac{3.32025 \times 10^{-16}}{5 \times 10^{-3}} = 6.6405 \times 10^{-14} = 664.05 \text{ pf}$$

$$C = 664.05 \text{ pf}$$

- (30) A parallel plate air core capacitor is floating with 10V across the plates. If the separation between the plates, is now increased to double the original value. Calculate the new value of the voltage across the capacitor.

Solution:

$$V = 10V$$

$$C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{t}$$

$$V = \frac{Q}{\epsilon_0 \epsilon_r A} \times t = 10 \text{ volts}$$

$$V_1 = \frac{Q}{\epsilon_0 \epsilon_r A} \times (2t)$$

$$V_1 = 2(10)$$

$$V_1 = 20 \text{ volts.}$$

Note: 2 is nothing but double

- (31) A parallel plate capacitor has a charge of 10^3 C on each plate while the potential difference between the plates is 1000V. Calculate the value of capacitance.

Solution:

$$Q = 10^3 = 1 \times 10^3 \text{ coulombs}$$

$$V = 1000 \text{ volts}$$

$$C = \frac{Q}{V} = \frac{1 \times 10^3}{1000} = 1 \times 10^{-6} = 1 \mu\text{f}$$

- (32) state the electric field intensity due to infinite line charge.

$$E = \frac{q}{2\pi \epsilon_0 \epsilon_r d} \text{ volts/meter}$$

where,

'q' → is charge density in coulombs per meter

'd' → is the perpendicular distance of the given point from line charge.

MAGNETOSTATICS

prepared by.

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SANCET

① STATE: Ampere's Circuital law.

Ampere's circuital law states that, the line integral of magnetic field intensity 'H' around any closed path equals the current enclosed by that path.

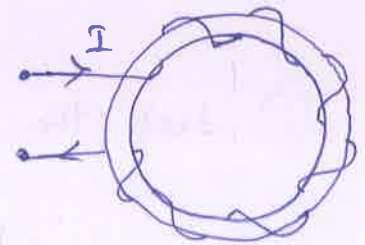
$$(i.e) \oint \vec{H} \cdot d\vec{l} = I_{(enclosed)}$$

② Apply Ampere's circuital law to calculate magnetic field intensity inside a current carrying toroid.

Let 'N' → Number of turns.

'R' → Mean radius of the toroid.

'H' → magnetic field intensity inside the toroid.



Apply Ampere's circuital law, and consider the flux path, we may write.

$$2\pi R \times H = NI$$

$$H = \frac{NI}{2\pi R}$$

③ A circular coil of radius of 2m carries a current of 4A what the value of magnetic field intensity at the centre.

$$I = 4A, R = 2m$$

$$H = \frac{NI}{2R} = \frac{1 \times 4}{2 \times 2} = 1 A/m.$$

(2)

④ Define scalar and vector magnetic potentials.

scalar potential: at any point inside a magnetic field is defined as $\frac{dW}{dm}$ Joules/weber.

where,

dW is work done in moving a test north pole of strength dm webers from infinity up to a given point.

vector potential: is defined as

$$\nabla \times \bar{A} = \bar{B}$$

where, \bar{A} is called vector magnetic potential.

⑤ List the Applications of Ampere's circuital law.

- (i) It is applied in magnetic circuit.
- (ii) It is applied in Maxwell's equation.

⑥ Write the expression for the magnetic field intensity at any point 'P' due to current-carrying conductor of (a) finite length & (b) infinite length.

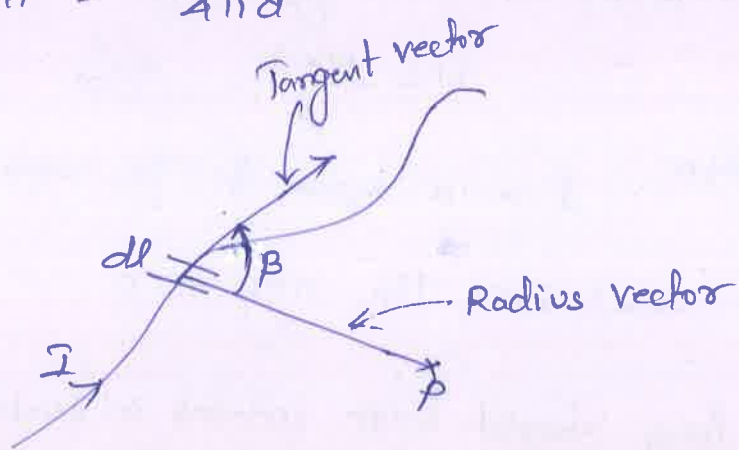
(a) The formula is $H = \left[\frac{I}{4\pi h} \right] [\cos\beta_1 - \cos\beta_2] \text{ A/m}$

(b) The formula is $H = \frac{I}{2\pi h} \text{ A/m}$

7 STATE BIOT-SAVART'S LAW.

According to Biot-Savart's law states that the differential magnetic field intensity $d\vec{H}$ produced at a point 'P', as shown in figure, the differential current element 'Idl' is proportional to the product $I dl$ and the sine of the angle β between the element and the line joining 'P' to the element and is inversely proportional to the square of the distance 'd' between 'P' and the element.

$$d\vec{H} = \frac{I \cdot dl}{4\pi d^2} \cdot \sin \beta \cdot \vec{a}_n \text{ A/m}$$



where, \vec{a}_n is an unit vector having direction given by Right Hand cork screw rule.

8 state the boundary conditions of magnetic media.

We define magnetic boundary conditions as the conditions that (H) or (B) field must satisfy at the boundary between two different media

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_{r1}}{\mu_{r2}}$$

9) State Lorentz's Law.

Lorentz's law states that the force experienced by a charged particle moving inside a magnetic field is given by

$$\vec{F} = q (\vec{v} \times \vec{B}) \text{ newtons (or) } \vec{F} = BIL \sin\alpha \text{ newtons}$$

$$\vec{F} = q B v \sin\alpha \hat{n} \text{ newtons.}$$

10) What will the magnetic field of a toroidal solenoid (with 'N' turns, carrying current I) within the volume of the ring and outside the ring.

(i) within the ring magnetic field intensity.

$$H = \frac{N \times I}{l} \text{ A/m}$$

where, $l \rightarrow$ is equal to the mean length of the toroid.

(ii) outside the ring $H = 0$.

11) A long straight wire carries a current $I = 1$ amp. At what distance is the magnetic field $H = 1$ A/m.

$$\text{we know } H = \frac{I}{2\pi d} \text{ A/m}$$

$$1 = \frac{1}{2\pi d}$$

$$\therefore d = \frac{1}{2\pi \times 1} = 0.0796 \text{ metres.}$$

12) Write the expression for magnetic force when moving charge particle.

This force is known as Lorentz force.

$$\therefore \vec{F} = q \cdot (\vec{v} \times \vec{B}) \text{ newtons.}$$

13) A circular coil of radius of 2m carries a current 4A what is the value of magnetic field intensity at the centre.

$$I = 4A, R = 2m$$

$$H = \frac{NI}{2R} \times \frac{1 \times 4}{2 \times 2} = 1 \text{ Ampers/meters.}$$

14) Define magnetic flux density.

The magnetic flux density 'B' is related to the magnetic field intensity 'H' according to

$$B = \mu_0 H \text{ wb/m}^2 \text{ (or) Tesla.}$$

where,

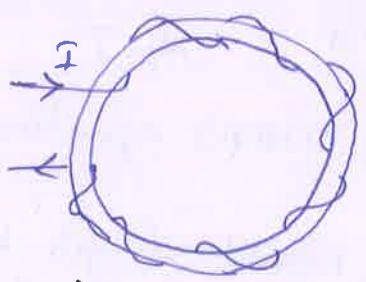
H → magnetic field intensity

μ_0 → is a constant known as the permeability of free space (air or vacuum).

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m.}$$

15) what is solenoid?

A solenoid is a cylindrically shaped coil consisting large number of turns wound on a non-magnetic frame. Then the flux through the solenoid is $\phi = BA$.



16) How magnetic materials are classified?

The magnetic materials can be classified according to their magnetic behavior, into.

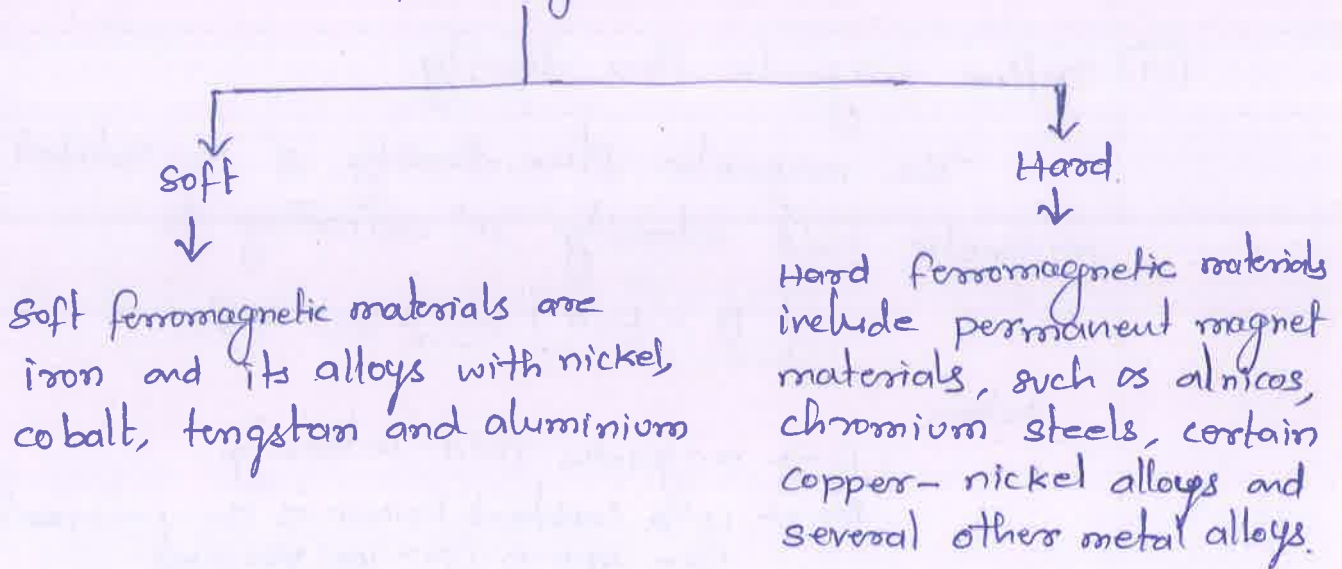
- (i) Diamagnetic
- (ii) paramagnetic
- (iii) ferromagnetic
- (iv) Antiferromagnetic
- (v) ferrimagnetic
- (vi) superparamagnetic.

17) Distinguish between diamagnetic, paramagnetic and ferromagnetic materials.

Diamagnetic, μ_r slightly less than 1

paramagnetic, μ_r slightly greater than 1

ferromagnetic materials.



note! silicon steels and cast steels are the most important ferromagnetic materials for use in the transformer & Electric machine.

18) write poisson's equation for magnetic field.

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

This is the poisson's equation for magnetostatic fields.

19) Determine force perunit length between two long parallel wires separated by 5cm in air and carrying current of 40 A in the same direction.

$$\frac{\text{Force (f)}}{\text{length (l)}} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi \times 10^{-7} \times 40 \times 40}{2\pi \times 5 \times 10^{-2}} = 6.4 \times 10^{-3} \text{ N/m}$$

20 Define Torque?

The torque on the current loop always tends to turn the loop so as to align the magnetic field produced by the loop with the applied magnetic field that causing torque.

$$T = IA \times B = m \times B$$

Where,

- T → Torque
- I → Current
- A → Area
- m → magnetic moment.

21 Give torque on a solenoid.

Torque on a solenoid in a magnetic field is

$$T = \frac{n}{2} \cdot 2IA B$$

$$T = nBI A$$

$$T = m B$$

where, $m = nIB$.

22 what is the expression for inductance of a toroid?

$$L = \frac{N^2 A \mu_0 \mu_r}{l}$$

$$L = \frac{N^2 \mu_0 \mu_r \pi r^2}{2\pi R}$$

$$L = \frac{N^2 \mu_0 \mu_r r^2}{2R}$$

- where,
- l → length of the Toroid $2\pi R$
 - A → Area of crosssection of steel ring
 - $A = \pi r^2$
 - r = radius of Toroid (rod)
 - R = Radius of mean Toroid

The purpose of the present study was to determine the effect of the use of a computer program on the learning of the multiplication table.

The study was conducted in a classroom of 20 students. The students were divided into two groups: a control group and an experimental group. The control group used traditional methods of learning the multiplication table, while the experimental group used a computer program.

The results of the study showed that the experimental group performed significantly better than the control group in terms of speed and accuracy in learning the multiplication table.

The following table shows the results of the study:

Group	Speed (minutes)	Accuracy (%)
Control	15	85
Experimental	10	95

The results of the study indicate that the use of a computer program is an effective method for learning the multiplication table. The program provides immediate feedback and allows for repeated practice, which helps to reinforce the learning.

The study has several limitations. First, the sample size was small, and the study was conducted in a single classroom. Future research should investigate the effectiveness of the program in larger classrooms and with a more diverse group of students.

TIME-Varying fields and MAXWELL'S EQUATION.

① state Faraday's law of electromagnetic induction.

According to this law,

⇒ whenever the magnetic flux linkages of the coil change with respect to time and an emf is induced in the coil.

⇒ The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux linkages.

$$\text{Mathematically } e = \frac{-d}{dt} (N\phi) \text{ volts.}$$

where,

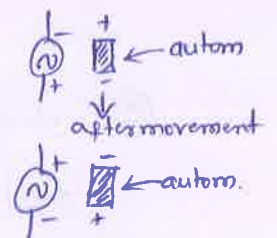
N = Number of turns in the circuit

e = Induced emf.

note! the minus sign indicates that the direction of the induced emf.

② Displacement current.

* A.C supply is given to the capacitor.



* So the plates of the capacitor will acquire positive charge and negative charge cyclically.

* As a result the dipoles inside rubber will get displaced

* As a result there will be current in the circuit.

* This current is due to displacement of dipoles.

$$\left. \begin{array}{l} \text{Displacement} \\ \text{current density} \end{array} \right\} \vec{J}_d = \frac{d\vec{D}}{dt} \text{ A/m}^2$$

$$\text{where, } D = \epsilon_0 \epsilon_r \vec{E}$$

3) conduction current.

conduction current is the current produced by the flow of free electrons as in the case of a copper wire.

$$\vec{J}_c = \sigma \vec{E}$$

where,

' σ ' is the conductivity of the medium.

' J_c ' is the conduction current density in A/m^2

4) state maxwell's I equation.

(or)

state maxwell's equation based on Ampere's law.

Ampere's law states that the line integral of magnetic field intensity H on any closed path is equal to the current enclosed by that path.

$$\oint_c \vec{H} \cdot d\vec{l} = I_{\text{(current enclosed)}} = \left(\vec{J}_c + \frac{dD}{dt} \right) ds$$

conduction current density $\sigma \vec{E} = \vec{J}_c$

Displacement current density $\vec{J}_d = \frac{dD}{dt}$ (5)

5) state maxwell's II equation.

(or)

state maxwell's equation based on faraday's law.

Faraday's law states that the electromotive force (emf) induced in a circuit is equal to the rate of decrease of the magnetic flux linkages the circuit.

$$\oint_c \vec{E} \cdot d\vec{l} = - \iint_s \frac{d\vec{B}}{dt} \cdot d\vec{s}$$

6) what is meant by continuity equation?

By adding conduction current \vec{J}_c and displacement current \vec{J}_d , is called continuity equation.

(i.e) $\vec{J} = \vec{J}_c + \vec{J}_d$

7) write down the wave equations for E and H in conducting medium.

$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \sigma \frac{\partial \vec{E}}{\partial t} = 0$
 $\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} + \mu \sigma \frac{\partial \vec{H}}{\partial t} = 0$

8) write down the wave equations for E and H in a non-dissipative (free space) medium.

$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$
 $\nabla^2 \vec{H} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0$

(or)

$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$
 $\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$

9) Define propagation constant.

The propagation constant (γ) is a complex number, and it is given by

$\gamma = \alpha + j\beta$

where,

- $\alpha \rightarrow$ is attenuation constant
- $\beta \rightarrow$ is phase constant
- $\gamma \Rightarrow \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

Part (a) of the question is done in part (a)

For the second part, we have to find the value of λ such that the matrix is invertible.

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \neq 0$$

For the matrix to be invertible, its determinant must be non-zero.

$$\begin{aligned} \text{VI} \quad & \frac{1}{1} \cdot \frac{15-12}{1} - \frac{2}{1} \cdot \frac{15-12}{1} + \frac{3}{1} \cdot \frac{15-12}{1} \\ & = \frac{15-12}{1} - \frac{30-24}{1} + \frac{45-36}{1} \\ & = 3 - 6 + 9 = 6 \end{aligned}$$

Since the determinant is non-zero, the matrix is invertible.

$$\begin{aligned} \text{VII} \quad & \frac{1}{1} \cdot \frac{15-12}{1} - \frac{2}{1} \cdot \frac{15-12}{1} + \frac{3}{1} \cdot \frac{15-12}{1} \\ & = 3 - 6 + 9 = 6 \end{aligned}$$

$$\begin{aligned} \text{VIII} \quad & \frac{1}{1} \cdot \frac{15-12}{1} - \frac{2}{1} \cdot \frac{15-12}{1} + \frac{3}{1} \cdot \frac{15-12}{1} \\ & = 3 - 6 + 9 = 6 \end{aligned}$$

Therefore, the matrix is invertible for all values of λ .

$$A^{-1} = \frac{1}{6} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

$$\begin{aligned} \text{IX} \quad & \frac{1}{1} \cdot \frac{15-12}{1} - \frac{2}{1} \cdot \frac{15-12}{1} + \frac{3}{1} \cdot \frac{15-12}{1} \\ & = 3 - 6 + 9 = 6 \end{aligned}$$

PLANE ELECTROMAGNETIC WAVES

① Define wave.

wave means at any point space \vec{E} or \vec{H} varies sinusoidally and also the electromagnetic energy travels through space at the velocity of light.

② Mention the properties of uniform plane wave.

(i) At every point in space, the electric field E and magnetic field H are perpendicular to each other and to the direction of the travel.

(ii) The field vary harmonically with time at the same frequency every where in space.

(iii) Each field has the same direction, magnitude and phase at every point in any plane perpendicular to the direction of wave travel.

③ what is Brewster angle?

Brewster angle is an incident angle at which there is no ~~reflection~~ reflected wave for parallelly polarized wave.

$$\theta = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

where,

ϵ_1 is dielectric constant of medium 1

ϵ_2 is dielectric constant of medium 2.

- ④ Define skin depth and its significant at low frequency and at high frequency application to conductors.

skin depth δ is defined as the depth in which the strength of the incident electromagnetic wave gets attenuated to $\frac{1}{e}$ times its original value, where $e = 2.718$.

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}} \text{ metres}$$

where,

$$\omega = 2\pi f$$

σ = is conductivity of medium

μ = permeability of medium.

Significance of δ is that at low frequency the value of δ is high whereas at high frequency the value of δ is small in the case of conductors.

- ⑤ state Poynting theorem.

According to Poynting theorem.

$$\iiint_V \vec{E} \cdot \vec{J} dV = -\frac{\partial}{\partial t} \iiint_V \left[\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right] dV - \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s}$$

\Rightarrow Poynting theorem states that the net power flowing out of given volume V is equal to the time rate of decrease in the energy stored within V minus the ohmic losses.

- ⑥ Define Poynting vector.

The vector product of E and H at any point is a measure of the rate of flow of energy per unit area at that point.

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \text{ watts/m}^2$$

* The direction of 'P' is perpendicular to E and H

* It measures the rate of flow of energy of the wave as it propagates.

⑦ what is the velocity of electromagnetic wave in free space and in lossless dielectric?

(i) In free space the velocity $v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ metres/sec

(ii) In lossless dielectric $v = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$ metres/sec

⑧ Find the velocity of plane wave in a lossless medium having a relative permittivity of 4 and relative permeability of 1.2.

we know the formula $v = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1.2 \times 8.854 \times 10^{-12} \times 4}}$$

$$v = 1.368 \times 10^8 \text{ metres/sec.}$$

⑨ Find the skin depth at a frequency of 2 MHz in aluminium where $\sigma = 38.2 \text{ MS/m}$ and $\mu_r = 1$

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}}$$

$$\delta = \sqrt{\frac{2}{2\pi f \times 38.2 \times 10^6 \times 4\pi \times 10^{-7} \times 1}}$$

$$\delta = 5.758 \times 10^{-5} \text{ metre.}$$

⑩ write down the expression for average power flow in electromagnetic field and average pointing vector.

(i) Average power $W_{av} = \frac{|V| |I|}{2} \cos \theta$

(ii) Average pointing vector $P_{av} = \frac{1}{2} \text{ Real part of } [E \times H^*]$

(11) Find the depth of penetration of a plane wave in copper at a power frequency of 60 Hz and at microwave frequency 10^{10} Hz. Given $\sigma = 5.8 \times 10^7$ mho/m.

$$\text{Depth of penetration } \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \sigma \mu}}$$

$$\delta = \sqrt{\frac{2}{2\pi \times 60 \times 5.8 \times 10^7 \times 4\pi \times 10^{-7}}}$$

$$\delta = 8.53 \times 10^{-3} \text{ m}$$